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Transverse spin asymmetries in Drell–Yan processes: $p^{\uparrow}p \to \mu^+\mu^- X$

E. Di Salvo

Dipartimento di Fisica and I.N.F.N., Sez. Genova, Via Dodecaneso, 33, 16146 Genova, Italy

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Abstract. We consider two different spin asymmetries in Drell–Yan processes generated by the collisions of an unpolarized proton beam on transversely polarized protons: the muon helicity asymmetry and the left–right asymmetry. We calculate the asymmetries in the framework of the QCD improved parton model, taking into account the parton transverse momentum and considering first-order QCD corrections. The muon helicity asymmetry is sensitive to the quark transversity distribution and is nonvanishing even at zero order. On the contrary the left–right asymmetry vanishes at zero order but not at first order in the QCD coupling constant, as a result of the gluon contribution.

1 Introduction

A major problem concerning the spin structure of the proton is the possibility of inferring the quark transversity distribution [1–5] from the data [6,7]. As pointed out by various authors [8–12], it is quite difficult to realize an experiment that is definitely sensitive to this quantity, which we shall denote by $h_1^f(x)$, f being the quark flavor. Indeed

(i) e–p deep inelastic scattering yields terms which are proportional to h_1^f but suppressed like m_f/Q, where m_f is the current quark mass and Q the mass of the virtual photon.
(ii) The double transverse spin asymmetry [5,13] in Drell–Yan processes is proportional to the product $h_1^f(x)\overline{h}_1^f(x')$ [11], which is probably very small, since we believe that, similarly to the helicity distributions, $|\overline{h}_1^f(x)| << |h_1^f(x)|$.
(iii) Other suggested [9,14] or planned [15] experiments are rather complicated and may present serious mishaps.

Up to now we can make only a rough evaluation of h_1^f [16], based on the recent results of seminclusive π electroproduction from HERMES and SMC [6,7]. Therefore doubts have been cast on the possibility of determining h_1^f [10]. In the present situation, the best we can do is to search for all possible spin asymmetries, which are related to the transversity distribution, and to try to extract as much as possible information from the data when these will be available. The aim of the present paper is to suggest a new experiment sensitive to h_1^f. We propose to observe a muon pair produced by a collision of a transversely polarized proton and an unpolarized one, detecting, by means of standard experimental techniques [17], the muon helicity asymmetry A_1 – that is, the average longitudinal polarization – of at least one of the muons (say μ^-). This kind of proposal is complementary to a typical double polarization experiment (e.g. inclusive DIS with polarized proton target and electron beam) and is somewhat analogous to Collins' idea [18], which consists in detecting the final quark polarization through the azimuthal asymmetry of the fragmentation function.

Spin asymmetries (especially with a single transverse spin) [19–33] are currently being considered in the literature [33], in view of experiments realized at FNAL [34] and DESY [6], or planned at RHIC [37–40] and other facilities [13,35]. The aim of such proposals is to obtain the magnitude of twist-three terms, from which the spin asymmetries are expected to generate a nonzero contribution. The calculation of such terms involves technical difficulties and not all authors agree on the final results [39]. Of particular interest is the left–right asymmetry [25,27], often called single transverse spin asymmetry, which can be determined from the Drell–Yan collision described above by detecting only the muon momenta. This asymmetry – which we call A_2 – is nonzero only if we consider, at least, the first-order correction in the QCD coupling constant g, for which the contribution of a soft gluon pole plays a major role. However, two different calculations [25,27] do not lead to the same formula.

We calculate the asymmetries A_1 and A_2 at tree level, assuming the QCD parton model [40] (see also [22]), and successively inserting "real" gluon corrections, for which we adopt an axial gauge [41,42], allowing us to develop a formalism as close as possible to the parton model. In calculating the gluon corrections, we introduce the quark–quark correlation functions, typical nonperturbative quantities. Our approach is different from the one by other authors [19,25,27] who adopt the formalism of quantum field theory. However, we match the two ways, obtaining a precise definition of the correlation function in terms of annihilation and creation operators. This suggests a general procedure for calculating higher twist contributions in the parton model.

Our results on A_2 substantially agree with those of [25]; however, the *gluonic pole* contribution is unfounded. In our approach the correlation functions are interpreted in terms of transverse quark and gluon polarization; moreover, they can be evaluated quantitatively within a model by Qiu and Sterman [19], which we extend in quite a natural way to the case of polarized correlation functions. The muon helicity asymmetry is strictly related to the transverse momentum dependent transversity distribution, whose integral over transverse momentum is h_1^f. Furthermore, we observe that neglecting transverse momentum implies the vanishing of the average helicity of the final muons, as a consequence of parity conservation. In this connection we like to stress the importance of the transverse momentum of the partons in various asymmetries [22].

The Drell–Yan cross section is generally complicated [42–44]; it becomes considerably simpler [38,45] – and in particular it may be approximated by a convolution over transverse momentum [46,47] – if we limit ourselves to the relatively large transverse momenta of the muon pairs [47]. This limitation is not a serious obstacle for our aims and it is even particularly appropriate for determining h_1^f, as we shall see.

In Sect. 2 we define the two asymmetries we want to calculate, and we give some general formulae for the asymmetries, the Drell–Yan cross section and the expression of the leptonic tensor. Section 3 is dedicated to the calculation of the hadronic tensor in the QCD parton model. In Sect. 4 we write the QCD first-order corrections by adopting an axial gauge. Moreover, we impose gauge invariance and establish some symmetry properties for the correlation functions. Then we illustrate the properties of the perturbatively calculable ("hard") coefficients; lastly we perform the calculations. In Sect. 5 we give the expressions of the asymmetries and evaluate their orders of magnitude. In Sect. 6 we draw some short conclusions.

2 General formulae

In Drell–Yan processes generated by the collisions between an unpolarized proton beam and a transversely polarized one, $p^\uparrow p \to \mu^+\mu^- X$, two different kinds of asymmetries can be defined, i.e., the muon helicity asymmetry if the polarization of one final muon is detected, and the left–right asymmetry if only the final momenta of the muons are determined.

(i) The muon helicity asymmetry is defined as

$$A_1 = \frac{\mathrm{d}\sigma_+ - \mathrm{d}\sigma_-}{\mathrm{d}\sigma_+ + \mathrm{d}\sigma_-}, \tag{1}$$

where $\mathrm{d}\sigma_\pm$ is the inclusive Drell–Yan differential cross section with the final μ^- having positive (negative) helicity.

(ii) The left–right asymmetry is

$$A_2 = \frac{\mathrm{d}\sigma_r - \mathrm{d}\sigma_l}{\mathrm{d}\sigma_r + \mathrm{d}\sigma_l}, \tag{2}$$

where $\mathrm{d}\sigma_{\mathrm{r(l)}}$ is the differential cross section with a final μ^- at the right (left) of the plane determined by the momentum and by the spin of the polarized proton. Indeed, if the muon helicities are not detected, the only way of constructing an asymmetric term (i.e., containing the ε tensor) is

$$A_2 \propto \overline{S} \cdot p, \tag{3}$$

where p is the μ^- four-momentum and

$$\overline{S}_\alpha = \varepsilon_{\alpha\beta\gamma\delta} n_1^\beta n_2^\gamma S^\delta, \tag{4}$$

S being the Pauli–Lubanski four-vector of the polarized proton. Here we also have set

$$n_{1(2)} \simeq \frac{1}{\sqrt{2}} \frac{P_{1(2)}}{|\boldsymbol{P}_{1(2)}|}, \tag{5}$$

and P_1 and P_2 are the four-momenta of the two protons, whose spatial parts are $\boldsymbol{P}_1$ and $\boldsymbol{P}_2$. Proton 1 is polarized, whereas 2 is not. In the laboratory frame, where $\boldsymbol{P}_1 = -\boldsymbol{P}_2$, (3) implies that the number of μ^- falling at the right of the plane determined by the spin and momentum of proton 1 is different from those which occur at the left of that plane.

The Drell–Yan cross section reads

$$\mathrm{d}\sigma = \frac{1}{4P\sqrt{s}} \frac{e^4}{Q^4} L_{\mu\nu} H^{\mu\nu} \mathrm{d}\Gamma, \tag{6}$$

where $\mathrm{d}\Gamma$ is the phase-space element, whose expression is

$$\mathrm{d}\Gamma = \frac{1}{(2\pi)^2} \mathrm{d}^4 p \delta(p^2)\theta(p_0) \mathrm{d}^4\overline{p}\delta(\overline{p}^2)\theta(\overline{p}_0)\delta^4(p+\overline{p}-q). \tag{7}$$

Here p and $\overline{p}$ are the four-momenta of μ^+ and μ^-, respectively, q is the four-momentum of the virtual photon, Q its effective mass, $P = |\boldsymbol{P}_1| = |\boldsymbol{P}_2|$ the modulus of the momentum of each proton in the laboratory frame, and s the overall energy square; $L_{\mu\nu}$ is the leptonic tensor and $H_{\mu\nu}$ the hadronic tensor.

The leptonic tensor is, in the massless approximation,

$$L_{\mu\nu} = \frac{1}{4} \mathrm{Tr}[\not{p}(1 \pm \gamma_5)\gamma_\mu \not{\overline{p}} \gamma_\nu], \tag{8}$$

$\pm$ being the sign of the μ^- helicity. Then we can write

$$L_{\mu\nu} = S^l_{\mu\nu} \pm \mathrm{i} A^l_{\mu\nu}, \tag{9}$$

where $S^l_{\mu\nu}$ and $\mathrm{i}A^l_{\mu\nu}$ are, respectively, the symmetric (real) part and the antisymmetric (imaginary) part of the leptonic tensor:

$$S^l_{\mu\nu} = p_\mu \overline{p}_\nu + \overline{p}_\mu p_\nu - g_{\mu\nu} p \cdot \overline{p}, \tag{10}$$

$$A^l_{\mu\nu} = \varepsilon_{\alpha\mu\beta\nu} p^\alpha \overline{p}^\beta. \tag{11}$$

Concerning the hadronic tensor, we set

$$H_{\mu\nu} = H^{(0)}_{\mu\nu} + H'_{\mu\nu}, \tag{12}$$

where $H^{(0)}_{\mu\nu}$ and $H'_{\mu\nu}$ are, respectively, the QCD zero-order and first-order approximations, which will be calculated in the next two sections.

3 Drell–Yan hadronic tensor in the QCD zero-order parton model

Now we write the hadronic tensor for the high energy Drell–Yan process. According to the QCD parton model [40], we take into account the transverse momentum of the quark inside the hadrons. Moreover, as already explained in the introduction, we limit ourselves to time-like photons with transverse momenta $|\boldsymbol{q}_\perp|$ of order 1 GeV and with $Q \gg |\boldsymbol{q}_\perp|$, where $Q^2 = q^2$ and q is the four-momentum of the pair. Then the generalized factorization theorem [19] in the covariant formalism [48] yields

$$H^{(0)}_{\mu\nu} = \sum_{f=1}^{3} e_f^2 H^f_{\mu\nu}, \tag{13}$$

where f are the three light flavors (u, d, s), $e_1 = 2/3$, $e_2 = e_3 = -1/3$, and

$$H^f_{\mu\nu} = \int \mathrm{d}\Gamma_q \sum_{T_1,T_2} [q^f_{T_1}(p_1)\overline{q}^f_{T_2}(p_2) h^{\tilde{T}_{12}}_{\mu\nu}(p_1,p_2;S) + (1 \leftrightarrow 2)]. \tag{14}$$

Here $q^f_{T_l}$ ($\overline{q}^f_{T_l}$) ($l = 1,2$) are the probability density functions of finding a quark (antiquark) in a pure spin state whose third component along the proton spin is T_l. Moreover, $\mathrm{d}\Gamma_q$ is the analog of the element of phase space of muons (see (7)):

$$\mathrm{d}\Gamma_q = \frac{1}{(2\pi)^2}\mathrm{d}^4p_1\delta(p_1^2)\theta(p_{10})\mathrm{d}^4p_2\delta(p_2^2)\theta(p_{20}) \times \delta^4(p_1+p_2-q), \tag{15}$$

and p_l are the four-momenta of the two active partons. These are taken on shell and massless, which is a good approximation for the values of $|\boldsymbol{q}_\perp|$ considered. Lastly

$$h^{\tilde{T}_{12}}_{\mu\nu} = \frac{1}{3}\mathrm{Tr}(\rho^{T_1}\gamma_\mu\overline{\rho}^{T_2}\gamma_\nu), \tag{16}$$

where the factor 1/3 comes from color averaging in q–$\overline{q}$ annihilation, $\tilde{T}_{12} \equiv (T_1, T_2)$ and ρ is the spin density matrix [49],

$$\rho^{T_l}(\overline{\rho}^{T_l}) = \frac{1}{2}\not{p}_l[1 + 2T_l\gamma_5(\pm\eta_\parallel + \not{\eta}_\perp)]. \tag{17}$$

$2T_l\eta_\parallel$ and $2T_l\eta_\perp$ are, respectively, the helicity and the transverse Pauli–Lubanski four-vector of the active partons. We have $\eta_\parallel = \boldsymbol{S}\cdot\boldsymbol{n}_l$, where $\boldsymbol{n}_l = \boldsymbol{p}_l/|\boldsymbol{p}_l|$ and $\boldsymbol{S}$ and $\boldsymbol{p}_l$ are, respectively, the space components of S and of p_l in the laboratory frame. Moreover, $\eta_\perp \equiv (0, \boldsymbol{S} - \eta_\parallel\boldsymbol{n}_l)$ ($l = 1,2$) and $\eta_\parallel$, a Lorentz scalar in the limit of zero quark mass, can be defined covariantly [50].

Carrying on the integration (14) over the time and longitudinal components of p_1, and adopting the light cone formalism, we get, in the limit of high Q^2 and $|\boldsymbol{p}_{l\perp}| \ll Q$ ($l = 1,2$),

$$H^f_{\mu\nu} = \frac{1}{4\pi^2Q^2}\int \mathrm{d}^2p_{1\perp}\sum_{T_1,T_2}[q^f_{T_1}(x_1,\boldsymbol{p}_{1\perp})\overline{q}^f_{T_2}(x_2,\boldsymbol{p}_{2\perp}) \times h^{\tilde{T}_{12}}_{\mu\nu}(x_1,x_2;S) + (1\leftrightarrow 2)]. \tag{18}$$

Here $x_{1,2} = (q_0 \pm q_\parallel)/s^{1/2}$ are the longitudinal fractional momenta of the two active partons, q_0 and $q_\parallel$ are, respectively, the time and longitudinal component of q, and

$$\boldsymbol{p}_{2\perp} = \boldsymbol{q}_\perp - \boldsymbol{p}_{1\perp}. \tag{19}$$

Since proton 2 is unpolarized, we set

$$q^f_{(T_2=1/2)} = q^f_{(T_2=-1/2)}, \quad \overline{q}^f_{(T_2=1/2)} = \overline{q}^f_{(T_2=-1/2)}. \tag{20}$$

Inserting (16) to (20) into (14), and taking into account the relation

$$Q^2 = 4x_1x_2P^2, \tag{21}$$

we get

$$H^f_{\mu\nu} = S^f_{\mu\nu} + \mathrm{i}A^f_{\mu\nu}, \tag{22}$$

where $S^f_{\mu\nu}$ and $\mathrm{i}A^f_{\mu\nu}$ are, respectively, the symmetric and antisymmetric part of the hadronic tensor:

$$S^f_{\mu\nu} = \frac{1}{24\pi^2}s_{\mu\nu}Q^f, \tag{23}$$

$$A^f_{\mu\nu} = \frac{2\sqrt{x_1x_2}}{24\pi^2Q}a_{\mu\nu}\delta Q^f, \tag{24}$$

where

$$Q^f = \int \mathrm{d}^2p_{1\perp}[q^f_1(x_1,\boldsymbol{p}^2_{1\perp})\overline{q}^f_2(x_2,\boldsymbol{p}^2_{2\perp}) + (1\leftrightarrow 2)], \tag{25}$$

$$\delta Q^f = \int \mathrm{d}^2p_{1\perp}\frac{\boldsymbol{S}\cdot\boldsymbol{p}_{1\perp}}{x_1}[\delta q^f_1(x_1,\boldsymbol{p}_{1\perp})\overline{q}^f_2(x_2,\boldsymbol{p}^2_{2\perp}) - \delta\overline{q}^f_1(x_1,\boldsymbol{p}_{1\perp})q^f_2(x_2,\boldsymbol{p}^2_{2\perp})]. \tag{26}$$

Here we have introduced the distribution functions:

$$q^f_l = \sum_{T_l=-1/2}^{1/2} q^f_{T_l}, \quad \delta q^f_l = \sum_{T_l=-1/2}^{1/2} 2T_l q^f_{T_l}, \tag{27}$$

and similarly for the antiquarks. Furthermore,

$$s_{\mu\nu} = n_{1\mu}n_{2\nu} + n_{2\mu}n_{1\nu} - g_{\mu\nu}, \quad a_{\mu\nu} = \varepsilon_{\alpha\mu\beta\nu}n_1^\alpha n_2^\beta. \tag{28}$$

Such tensors fulfil gauge invariance up to twist-four terms. The distributions δq^f are related to the transversity functions h^f_1:

$$h^f_1(x) = \int \mathrm{d}^2p_\perp \delta q^f(x,\boldsymbol{p}_\perp), \tag{29}$$

a similar relation holding true for antiquarks.

Now we discuss a symmetry property of the spin density functions δq^f. Invariance of the strong interactions under parity inversion, time reversal and rotations (in particular rotations of π around the proton momentum) imply

$$\delta q^f(x, \boldsymbol{p}_\perp) = \delta q^f(x, -\boldsymbol{p}_\perp). \tag{30}$$

However, if we take into account the initial state interactions and soft gluon exchange between the protons, we have to introduce also the *effective*, T-odd, density functions [22], which do not fulfill (30), since time reversal invariance does not trivially apply to them. This fact has no particular consequences for the integral (26), which in general does not vanish for $|\boldsymbol{q}_\perp|$ of order 1 GeV. However, if we integrate the cross section over the transverse momentum of the muon pair, the antisymmetric hadronic tensor derives its contribution (if any) from the sole *effective* density functions.

Inserting (22) into (13), we get

$$H^{(0)}_{\mu\nu} = S^{(0)}_{\mu\nu} + \mathrm{i}A^{(0)}_{\mu\nu}, \tag{31}$$

where

$$S^{(0)}_{\mu\nu} = \sum_{f=1}^{3} e_f^2 S^f_{\mu\nu}, \quad A^{(0)}_{\mu\nu} = \sum_{f=1}^{3} e_f^2 A^f_{\mu\nu}. \tag{32}$$

In order to find the asymmetries A_1 and A_2 (see (1) and (2)), we combine the hadronic tensor (31) with the leptonic tensor (9), according to (6) for the differential cross section. Therefore, in the zero-order approximation of the QCD parton model, we find the following results:

(i) If the helicity of the final negative muon is detected, the leptonic tensor has a nonvanishing antisymmetric part, which, combined with the antisymmetric part of the hadronic tensor (31), gives a nonzero contribution to the muon helicity asymmetry A_1.

(ii) On the other hand, if no helicity is detected, only the symmetric part of the leptonic tensor survives and combination with the hadronic tensor drops out of $A^{(0)}_{\mu\nu}$. Since $S^f_{\mu\nu}$ (see (23)) is spin independent, the left–right asymmetry A_2 vanishes. This result may be also deduced from parity inversion and time reversal invariance.

Concerning the muon helicity asymmetry, we are faced with two important questions:

(1) The asymmetry A_1 and the transversity function h_1^f depend on the distribution function δq^f through integral relations; respectively, (26) and (29). Therefore, we have to prove that the function we extract from (26) contributes to h_1^f.

(2) One may wonder: where does a chiral-odd function like h_1^f come from in the process considered? Such a distribution function arises, e.g., in DY with two polarized proton beams, or from a mass term, which causes a helicity flip.

To answer the first question, we write the relation

$$\delta q^f = \cos\alpha(\delta q^f_{\mathrm{R}} - \delta q^f_{\mathrm{L}}) + \sin\alpha\delta q^f_H, \tag{33}$$

where

$$\delta q^f_{\mathrm{R(L)}} = |\langle q_{\mathrm{R(L)}}|P^\uparrow\rangle|^2, \tag{34}$$

$$\delta q^f = 2\mathrm{Re}[\langle P^\uparrow|q_{\mathrm{R}}\rangle\langle q_{\mathrm{L}}|P^\uparrow\rangle], \tag{35}$$

$$\cos\alpha = \frac{\boldsymbol{p}_T \cdot \boldsymbol{S}}{xP}. \tag{36}$$

Here $|P^\uparrow\rangle$ and $|q_{\mathrm{R(L)}}\rangle$ are shorthand notations for the canonical state of the proton and for quark helicity states. Relation (33) follows from decomposing into helicity states a quark state with a given transverse momentum and spin parallel or antiparallel to the proton spin. The first term of that relation, which is twist-three and chiral-even, may be expressed as a linear combination of the usual helicity distribution functions. On the contrary, the second term is chiral-odd and prevalently twist-two, since $\sin\alpha \sim 1$. Due to arguments similar to those which led to relation (30), we conclude that the first term of (33) does not contribute to h_1^f, as can be seen from (29). The transversity function depends solely on the chiral-odd function δq^f_H, which contributes to the muon helicity asymmetry, as can be checked by substituting (33) into (26).

As regards the origin of the chiral-odd function in our process, we show in Appendix A that the antisymmetric tensor (24) may be written as

$$A^f_{\mu\nu} = m_f \epsilon_{\alpha\mu\beta\nu} \int \mathrm{d}^2 p_{1\perp} \sum_{T_1,T_2} \Big[q^f_{T_1}(x_1, \boldsymbol{p}_{1\perp}) \bar{q}^f_{T_2}(x_2, \boldsymbol{p}^2_{2\perp}) \times S_f^{T_1\alpha} p_1^\beta - (1 \leftrightarrow 2) \Big]. \tag{37}$$

Here m_f and $S_f^{T_1}$ are, respectively, the mass and the Pauli–Lubanski four-vector of the active quark. The quark mass causes a helicity flip and therefore a contribution to the chiral-odd distribution function. The smallness of m_f is compensated by $S_f^{T_1}$, which results (see Appendix A) in

$$S_f^{T_1} \sim 2T_1 \boldsymbol{p}_{1\perp} \cdot \boldsymbol{S} \frac{\sqrt{2}n_1}{m_f x_1}. \tag{38}$$

The effect we have just illustrated is washed out in DIS, since we have to convolute the elementary quark–photon cross section over only one distribution function. In this case also the initial state interactions, and therefore the *effective* distribution functions, are suppressed.

4 QCD first-order contributions to spin asymmetries

Now we consider the q–$\bar{q}$ annihilation amplitudes with one “real” gluon emitted (absorbed) by one of the colliding hadrons and absorbed (emitted) by the active parton of the other hadron. These interfere with the amplitudes just considered in the preceding section. The interference terms are twist-three and first order in g. We adopt the axial gauge $A^a \cdot n_2 = 0$, where A^a is the gluonic field, $a = 1, ..., 8$ and n_2 is given by (5). This gauge is not covariant, but allows one to keep the parton model description; in particular, in our approximation, this gauge avoids the complication of "link" operators [52], which we should introduce in a generic gauge, in order to ensure gauge invariance of the nonperturbative ("soft") functions involved in scattering.

In this section we write the twist-three contributions according to the parton model [46]; secondly, we impose gauge invariance and deduce symmetry properties of the "soft" functions that we are going to introduce, the so-called correlation functions; thirdly we discuss the behavior of the fermionic propagator, which appears in the "hard" coefficient; lastly we perform the calculations.

4.1 Parton model approach

The twist-three contribution to the hadronic tensor reads, in tree approximation,

$$H'_{\mu\nu} = g \sum_{f=1}^{3} e_f^2 (H'^{f,a}_{\mu\nu} - H'^{f,b}_{\mu\nu}), \tag{39}$$

where

$$H'^{f,a}_{\mu\nu} = \sum_{l=1}^{2} H'^{f,a,l}_{\mu\nu}, \quad H'^{f,b}_{\mu\nu} = \sum_{l=1}^{2} H'^{f,b,l}_{\mu\nu}. \tag{40}$$

Each term in (40) corresponds to a different graph:
(i) $H'^{f,a,l}_{\mu\nu}$ ($l = 1, 2$) refer to a gluon emitted (or absorbed) by proton 1 – which is polarized – and absorbed (or emitted) by the active antiquark ($l = 1$) or quark ($l = 2$) of proton 2.
(ii) $H'^{f,b,l}_{\mu\nu}$ refer instead to a gluon emitted (or absorbed) by proton 2 and absorbed (or emitted) by the active parton of proton 1. Analytically we have

$$H'^{f,a,1}_{\mu\nu} = \int \mathrm{d}\Omega_1 \sum_{T_1,T'_1,T_2} c^{f,1}_{T_1,T'_1}(x_1, \boldsymbol{p}_{1\perp}; x'_1, \boldsymbol{p}'_{1\perp}) \times \bar{q}^f_{T_2}(x_2, \boldsymbol{p}_{2\perp}) h'^{\overline{T}_{12}}_{\mu\nu}(x_1, x'_1, x_2), \tag{41}$$

$$H'^{f,a,2}_{\mu\nu} = \int \mathrm{d}\Omega_1 \sum_{T_1,T'_1,T_2} \bar{c}^{f,1}_{T_1,T'_1}(x_1, \boldsymbol{p}_{1\perp}; x'_1, \boldsymbol{p}'_{1\perp}) \times q^f_{T_2}(x_2, \boldsymbol{p}_{2\perp}) \overline{h}'^{\overline{T}_{12}}_{\mu\nu}(x_1, x'_1, x_2), \tag{42}$$

while $H'^{f,b,l}_{\mu\nu}$ are obtained from (41) and (42) by substituting ($1 \leftrightarrow 2$) and $\mu \leftrightarrow \nu$. $c^{f,l}_{T_l,T'_l}$ ($\bar{c}^{f,l}_{T_l,T'_l}$) ($l = 1, 2$) is the correlation function between a quark (antiquark) of four-momentum p_l and spin T_l and another of four-momentum p'_l and spin T'_l. $d\Omega_l$ is the phase-space element, i.e.,

$$\mathrm{d}\Omega_l = \mathrm{d}\Gamma_q \frac{1}{(2\pi)^3} \mathrm{d}^4 k_l \delta(k_l^2) \mathrm{d}^4 p'_l \delta(p'^2_l) \times \delta^4(p_l - p'_l - k_l)\theta(p'_{l0}) \to \frac{1}{2(2\pi)^5 Q^2} \mathrm{d}^2 p_{l\perp} \mathrm{d}^2 k_{l\perp} \frac{\mathrm{d}z_l}{z_l}, \tag{43}$$

where $\mathrm{d}\Gamma_q$ is given by (15), k_l is the four-momentum of the absorbed (or emitted) gluon and $z_l = x'_l - x_l$. The arrow denotes integration over p_{l0}, $p_{l\parallel}$, k_{l0}, $k_{l\parallel}$. Lastly, in the axial gauge $A^a \cdot n_2 = 0$ we have

$$h'^{\overline{T}_{12}}_{\mu\nu} = \frac{1}{3} \frac{1}{\overline{p}_2^2 + \mathrm{i}\epsilon} Tr \left[\kappa^{T_1,T'_1} \gamma_\mu \left(\not{\overline{p}}_2 \not{e} \overline{\rho}^{T_2} + \overline{\rho}^{T_2} \not{e} \not{\overline{p}}_2 \right) \gamma_\nu \right], \tag{44}$$

where

$$\overline{T}_{12} \equiv (T_1, T'_1, T_2), \quad \overline{p}_2 = p_2 - k_1, \tag{45}$$

e_μ is the polarization four-vector of the gluon and $\kappa^{T,T'}$ the spin correlation matrix between a quark of spin T and another with spin T'. $\overline{h}'^{\overline{T}_{12}}_{\mu\nu}$ is obtained from (44) by changing the quark matrices with the antiquark matrices and vice versa, i.e., $\kappa^{T_1,T'_1} \to \overline{\kappa}^{T_1,T'_1}$ and $\overline{\rho}^{T_2} \to \rho^{T_2}$. As shown in Appendix B,

$$\kappa^{T,T'} = \psi(t) \kappa_p^{T,T'}, \quad \overline{\kappa}^{T,T'} = \psi(t) \overline{\kappa}_p^{T,T'}, \tag{46}$$

where

$$t = \frac{x - x'}{x + x'}, \quad \psi(t) = \frac{1 + t}{\sqrt{1 - t^2}}, \tag{47}$$

$$\kappa_p^{T,T} = \overline{\kappa}_p^{T,T} = \frac{1}{2} \not{p}(1 + 2T\gamma_5 \not{S}),$$
$$\kappa_p^{T,-T}(\overline{\kappa}_p^{T,-T}) = \frac{1}{2} \not{p}\gamma_5(\not{S} \mp 2\mathrm{i}T). \tag{48}$$

Since we are considering the twist-three contribution, we neglect the transverse momentum in the numerators of $h'^{\overline{T}}_{\mu\nu}$ and $\overline{h}'^{\overline{T}}_{\mu\nu}$. In this approximation the spin density matrix reads

$$\rho^T = \overline{\rho}^T = \frac{1}{2} \not{p}(1 + 2T\gamma_5 \not{S}). \tag{49}$$

In the next subsection we shall show that the sum $H^{(0)}_{\mu\nu} + H'_{\mu\nu}$ – where $H^{(0)}_{\mu\nu}$ is given by (13) and $H'_{\mu\nu}$ by (39) – is gauge invariant, although the single terms are not.

Since proton 1 is transversely polarized, quark–gluon interactions involve linearly polarized gluons [49], either in the direction of the spin of the proton, or in the direction orthogonal to the spin and to the momentum of the proton. In Appendix B we show that

$$T'_1 = T_1 : \ e = 2T_2 \overline{S}; \quad T'_2 = T_2 : \ e = 2T_1 \overline{S}; \tag{50}$$
$$T'_l \neq T_l : \ e = -S\delta_{T_1,T_2}. \tag{51}$$

Substituting (46) and (48) to (51) into (44) and performing the calculations, we get

$$h'^{\overline{T}_{12}}_{\mu\nu} = h'^{\tilde{T}_{12}}_{N\mu\nu} \delta_{T'_1,T_1} + h'^{\tilde{T}_{12}}_{F\mu\nu} \delta_{T'_1,-T_1}, \tag{52}$$

where $\tilde{T}_{12} \equiv (T_1, T_2)$ and

$$h'^{\tilde{T}_{12}}_{\mathrm{N}\mu\nu} = \frac{1}{3} \psi(t_1) \frac{2p_2 \cdot k_1}{\overline{p}_2^2 + \mathrm{i}\epsilon} 2(T_1 + T_2)(p_{1\mu}\overline{S}_\nu + p_{1\nu}\overline{S}_\mu), \tag{53}$$

$$h'^{\tilde{T}_{12}}_{\mathrm{F}\mu\nu} = \frac{1}{3}\psi(t_1)\frac{2p_2\cdot k_1}{\overline{p}_2^2+\mathrm{i}\epsilon}\left[2T_1\epsilon_{\alpha\mu\beta\nu}S^\alpha p_1^\beta \right. \\ \left. +\ 2T_2(p_{1\mu}\overline{S}_\nu + p_{1\nu}\overline{S}_\mu)\right]\delta_{T_1,T_2}. \quad (54)$$

Here the suffixes N and F denote, respectively, spin nonflip and flip of the active quark. The tensors $\overline{h}'^{\tilde{T}_{12}}_{\mathrm{N(F)}\mu\nu}$ are equal to the expressions (53) and (54), except for a change of sign in front of the first term of (54). $h'^{\tilde{T}_{21}}_{\mathrm{N(F)}\mu\nu}$ and $\overline{h}'^{\tilde{T}_{21}}_{\mathrm{N(F)}\mu\nu}$, involved in the expressions of $H'^{f,b,l}_{\mu\nu}$, are obtained from $h'^{\tilde{T}_{12}}_{\mathrm{N(F)}\mu\nu}$ and $\overline{h}'^{\tilde{T}_{12}}_{\mathrm{N(F)}\mu\nu}$ by substituting $(1 \leftrightarrow 2)$ and $\mu \leftrightarrow \nu$. Now we establish some important relations, which considerably simplify our calculations.

4.2 Correlation functions: normalization and symmetry properties

In Appendix C we show that gauge invariance implies the following normalization for the quark correlation functions:

$$c^{f,l}_{T_l,T'_l}(x_l,\boldsymbol{p}_{l\perp};x'_l,\boldsymbol{p}'_{l\perp})\delta^4(p_l-k_l-p'_l) \\ = 8\pi^{3/2}(|\boldsymbol{p}_l||\boldsymbol{k}_l||\boldsymbol{p}'_l|)^{1/2} \\ \times\sum_{i,j=1}^{3}\sum_{c=1}^{8}\langle P_l|c^{f\dagger}_{T'_l,i}(\boldsymbol{p}'_l)a^c_m(\boldsymbol{k}_l)c^f_{T_l,j}(\boldsymbol{p}_l)|P_l\rangle. \quad (55)$$

Here i, j and c are the color indices of the quarks and of the gluon, combined in such a way that the operator product constitutes a color singlet. $c^f_{T,i}$ and $c^{f\dagger}_{T,i}$ are the annihilation and creation operators for the quarks, $a^c_m(\boldsymbol{k}_l)(m=1,2)$ are annihilation operators for gluons whose polarization four-vector is S ($m=1$, corresponding to $T'_l \neq T_l$), or $\overline{S}$ ($m=2$, corresponding to $T'_l = T_l$). $a^c_m(\boldsymbol{k}_l)$ has to be substituted by $a^{c\dagger}_m(\boldsymbol{k}_l)$ for negative values of the gluon energy.

Invariance under parity inversion, time reversal, and rotation by π around the proton momentum imply, through (55),

$$c^{f,l}_{T'_l,T_l}(x'_l,\boldsymbol{p}'_{l\perp};x_l,\boldsymbol{p}_{l\perp}) = -c^{f,l}_{T_l,T'_l}(x_l,-\boldsymbol{p}_{l\perp};x'_l,-\boldsymbol{p}'_{l\perp}). \quad (56)$$

To show this, we observe that the combined action of the three transformations leaves spin and longitudinal momenta unchanged, while inverting the arguments of the function and transverse momenta. Moreover, time reversal produces the same change of phase ($n\pi$, with n integer) as the rotation. Therefore the minus sign in front the r.h.s. of (56) is due to the product of the intrinsic parities of the two fermions and of the gluon.

Furthermore, the hermiticity condition yields

$$c^{f,l}_{T'_l,T_l}(x'_l,\boldsymbol{p}'_{l\perp};x_l,\boldsymbol{p}_{l\perp}) = c^{f,l*}_{T_l,T'_l}(x_l,\boldsymbol{p}_{l\perp};x'_l,\boldsymbol{p}'_{l\perp}). \quad (57)$$

Therefore, if we integrate over transverse momenta, the correlation functions are imaginary and antisymmetric under the simultaneous exchange of the momenta and spin of the two quarks. Relations similar to (55)–(57) hold true for the antiquark correlation functions. Lastly, since proton 2 is unpolarized, we have

$$c^{f,2}_{T_2,T'_2} = c^{f,2}_{-T_2,-T'_2}, \quad \overline{c}^{f,2}_{T_2,T'_2} = \overline{c}^{f,2}_{-T_2,-T'_2}. \quad (58)$$

These equations, together with (20), imply that the terms proportional to T_2 in (53) and (54) (or in those which are obtained by substituting $(1 \leftrightarrow 2)$) are dropped after summing over the spin indices.

The symmetry property (56) does not hold if we take into account the initial state interaction and soft gluon exchange between the protons. Indeed, analogously to the *effective* density functions, it makes sense to introduce *effective*, T-odd, correlation functions, for which time reversal invariance cannot be expressed in a trivial way [22]. Therefore such functions – whose importance will be shown in the following subsection – have in general a real part even after integration over the transverse momentum.

4.3 The fermionic propagator

In twist-three approximation the tensor (44) – the "hard" coefficient of the hadronic tensor – is independent of the transverse momentum of the active parton. Therefore our preceding considerations imply that, aside from the *effective* correlation functions, the left–right asymmetry of the cross section *integrated* over the transverse momentum of the muon pair receives a contribution from first-order corrections only if the tensor (44) has an imaginary part. This is in agreement with the observation by Boer et al. [25], although we arrive at a different conclusion. We examine this question in detail.

The tensor (44) includes the factor $(2k_1\cdot p_2)/(\overline{p}_2^2+\mathrm{i}\epsilon)$. For a massless quark $\overline{p}_2^2 = -2k_1\cdot p_2$, which annihilates the effects of the imaginary part of the tensor, unless the correlation function $c^{f,l}_{T_l,T'_l}$ has a simple pole – a *gluonic pole* [25] – just at $\overline{p}_2^2 = 0$. If we would neglect, like Boer et al. [25], the transverse momentum effects in the fraction above, $\overline{p}_2^2$ would vanish at $z_1 = 0$ and it would make sense to assume a *gluonic pole* just at zero momentum (a soft gluon contribution), so that the fermion propagator would yield an imaginary part. But, at least in the approximation of on-shell and massless quarks – adopted also by Boer et al. [25] –, the transverse momentum has dramatic consequences on the pole, which as a result is found to be located at

$$z_1 = \frac{x_2P\boldsymbol{p}_{2\perp}\cdot\boldsymbol{k}_{1\perp} \pm \sqrt{\Delta}}{P\boldsymbol{p}^2_{2\perp}}, \\ \Delta = (x_2^2P^2+\boldsymbol{p}^2_{2\perp})[(\boldsymbol{p}_{2\perp}\cdot\boldsymbol{k}_{1\perp})^2 - \boldsymbol{p}^2_{2\perp}\boldsymbol{k}^2_{1\perp}]. \quad (59)$$

This value is generally complex, and by no means can be approximated by $z_1 \simeq 0$. Consequently a soft *gluonic pole* would not imply an imaginary part for the tensor (44). On the other hand, it appears quite arbitrary to assume that $c^{f,2}_{TT'}$ has a pole located just in correspondence with the value given by the first equation (59). Analogous considerations can be made for the pole $(\overline{p}_1^2)^{-1}$, with $\overline{p}_1 = p_1 - k_2$.

Therefore the "hard" coefficient of the hadronic tensor does not have an imaginary part. We just set the fractions in front of the tensors (53) and (54) equal to −1. As a consequence the twist-three contribution (if any) to the left–right asymmetry of the *integrated* cross section is entirely due to the *effective* correlation functions. Our considerations are somewhat analogous to those of Anselmino et al. [22], concerning pion inclusive production.

4.4 Calculation of the gluon correction

Now we perform the calculation of the hadronic tensor (39). Substituting (41) to (43) into (40), and taking into account the results (53) and (54) and the considerations above, we get

$$H'^{f,a}_{\mu\nu} = \frac{\sqrt{2}x_1 P}{6(2\pi)^5 Q^2}(s_{1\mu\nu}\tilde{\delta}_1 Q^f + \mathrm{i}a_{1\mu\nu}\hat{\delta}_1 Q^f), \quad (60)$$

$$H'^{f,b}_{\mu\nu} = \frac{\sqrt{2}x_2 P}{6(2\pi)^5 Q^2}(s_{2\mu\nu}\tilde{\delta}_2 Q^f + \mathrm{i}a_{2\mu\nu}\hat{\delta}_2 Q^f). \quad (61)$$

Here we have set

$$\begin{aligned}\tilde{\delta}_1 Q^f &= \int \mathrm{d}^2 p_{1\perp}[\delta C^f_{S,1}(x_1,\boldsymbol{p}_{1\perp})\overline{q}^f_2(x_2,\boldsymbol{p}^2_{2\perp}) \\ &\quad + \delta\overline{C}^f_{S,1}(x_1,\boldsymbol{p}_{1\perp})q^f_2(x_2,\boldsymbol{p}^2_{2\perp})], \quad (62)\\ \tilde{\delta}_2 Q^f &= \int \mathrm{d}^2 p_{1\perp}[C^f_{S,2}(x_2,\boldsymbol{p}_{2\perp})\delta\overline{q}^f_1(x_1,\boldsymbol{p}_{1\perp}) \\ &\quad + \overline{C}^f_{S,2}(x_2,\boldsymbol{p}_{2\perp})\delta q^f_1(x_1,\boldsymbol{p}_{1\perp})], \quad (63)\\ \hat{\delta}_1 Q^f &= \int \mathrm{d}^2 p_{1\perp}[\delta C^f_{A,1}(x_1,\boldsymbol{p}_{1\perp})\overline{q}^f_2(x_2,\boldsymbol{p}^2_{2\perp}) \\ &\quad - \delta\overline{C}^f_{A,1}(x_1,\boldsymbol{p}_{1\perp})q^f_2(x_2,\boldsymbol{p}^2_{2\perp})], \quad (64)\\ \hat{\delta}_2 Q^f &= \int \mathrm{d}^2 p_{1\perp}[C^f_{A,2}(x_2,\boldsymbol{p}_{2\perp})\delta\overline{q}^f_1(x_1,\boldsymbol{p}_{1\perp}) \\ &\quad - \overline{C}^f_{A,2}(x_2,\boldsymbol{p}_{2\perp})\delta q^f_1(x_1,\boldsymbol{p}_{1\perp})] \quad (65)\end{aligned}$$

and

$$s_{l\mu\nu} = \overline{S}_\mu n_{l\nu} + \overline{S}_\nu n_{l\mu}, \quad a_{l\mu\nu} = \epsilon_{\alpha\mu\beta\nu}S^\alpha n^\beta_l \quad (l=1,2). \quad (66)$$

Moreover,

$$\delta C^f_{S,1}(x_1,\boldsymbol{p}_{1\perp}) = \sum_T 2T C^{f,1}_{S,T}(x_1,\boldsymbol{p}_{1\perp}), \quad (67)$$

$$C^f_{S,2}(x_2,\boldsymbol{p}_{2\perp}) = \sum_T C^{f,2}_{S,T}(x_2,\boldsymbol{p}_{2\perp}), \quad (68)$$

$$\delta C^f_{A,1}(x_1,\boldsymbol{p}_{1\perp}) = \sum_T 2T C^{f,1}_{A,T}(x_1,\boldsymbol{p}_{1\perp}), \quad (69)$$

$$C^f_{A,2}(x_2,\boldsymbol{p}_{2\perp}) = \sum_T C^{f,2}_{A,T}(x_2,\boldsymbol{p}_{2\perp}), \quad (70)$$

where

$$C^{f,l}_{S,T}((x_l,\boldsymbol{p}_{l\perp}) = \mathrm{Re}\left\{\int \mathrm{d}^2 k_{l\perp}\int\frac{\mathrm{d}z_l}{z_l} \quad (71)\right.$$

$$\left.\times\left[\sum_{T'} c^{f,l}_{T,T'}(x_l,\boldsymbol{p}_{l\perp};x'_l,\boldsymbol{p}'_{l\perp})\right]\psi(t_l)\right\},$$

$$C^{f,l}_{A,T}((x_l,\boldsymbol{p}_{l\perp}) = \mathrm{Im}\left\{\int \mathrm{d}^2 k_{l\perp}\int\frac{\mathrm{d}z_l}{z_l} \times\left[c^{f,l}_{T,-T}(x_l,\boldsymbol{p}_{l\perp};x'_l,\boldsymbol{p}'_{l\perp})\right]\psi(t_l)\right\}, \quad (72)$$

and $l=1,2$. Analogous expressions hold for the barred quantities. Here z and $\boldsymbol{k}_\perp$ are defined in the expression (43) of the phase-space element, while t is given by the first equation (47). The QCD first-order correction $H'_{\mu\nu}$ is obtained substituting (60) and (61) into (39)). Three remarks are in order.

(1) The functions C^f_S, δC^f_S, C^f_A and δC^f_A have the dimensions of a mass, as can be checked from (55).
(2) Integrating the cross section over the transverse momentum of the muon pair amounts to independently integrating over the quark transverse momentum the "soft" functions involved in the hadronic tensor. In particular, as stressed in the previous subsection, the integrals of the functions $\tilde{\delta}_l Q^f$ ($l=1,2$, see (62) and (63)) derive their contributions solely from the *effective* correlation functions.
(3) Taking into account (47), the integrals (71) and (72) can be split into two terms, corresponding to

$$\frac{1}{2}[\tilde{c}^f(x,x+z)+\tilde{c}^f(x,x-z)]$$

and

$$\frac{1}{2z}[c^f(x,x+z)-c^f(x,x-z)], \quad (73)$$

where $c^f = c^{f,l}_{T,T'}(1-t^2)^{-1/2}$, $\tilde{c}^f = t/zc^f$ and t is given by the first (47). If we assume $c^f(x,x') = c^f_0(x,x')$ $\delta(x-x')$, where c^f_0 is a smooth function of its arguments, the second term (73) corresponds to the *derivative* term by Qiu and Sterman [19,20] (see [27] for details).

5 Asymmetries

In QCD first-order approximation the hadronic tensor $H_{\mu\nu}$ consists, according to (12), of the sum of two terms, which have been calculated in the two previous sections, (31) and (39). Substituting the expressions of $H_{\mu\nu}$ and of the leptonic tensor (9) into the differential cross section (6), and taking into account the (1) and (2) of the two asymmetries, we get

$$A_1 = A^{(0)}_1 + A^{(1)}_1, \quad (74)$$

where

$$A^{(0)}_1 = \frac{8\sqrt{x_1x_2}}{Q}\frac{\sum_f e^2_f\delta Q^f}{\sum_f e^2_f\ Q^f}\frac{\cos\theta}{1+\cos^2\theta} \quad (75)$$

and

$$A_1^{(1)} = \frac{-g}{(2\pi)^3\sqrt{x_1x_2}Q}\frac{\sum_f e_f^2(x_1\hat{\delta}_1 Q^f + x_2\hat{\delta}_2 Q^f)}{\sum_f e_f^2 Q^f} \times \frac{2\sin\theta\sin\phi}{1+\cos^2\theta}. \tag{76}$$

Moreover,

$$A_2 = \frac{g}{(2\pi)^3\sqrt{x_1x_2}Q}\frac{\sum_f e_f^2(x_1\tilde{\delta}_1 Q^f + x_2\tilde{\delta}_2 Q^f)}{\sum_f e_f^2 Q^f} \times \frac{\sin 2\theta\cos\phi}{1+\cos^2\theta}. \tag{77}$$

Here θ is the polar angle and ϕ the azimuthal angle of the negative muon: we have assumed a reference frame in the center-of-mass system of the muon pair, whose z-axis is taken along the momentum of the polarized proton, while the x-axis is along the space component of $\overline{S}$. Furthermore, $Q^f, \delta Q^f, \tilde{\delta}_l Q^f, \hat{\delta}_l Q^f$ $(l = 1,2)$ are given, respectively, by (25), (26) and (62) to (65).

The expression of A_2 turns out to coincide with the one by Boer et al. [25,52], provided

– we integrate the cross section over the transverse momentum of the virtual photon;
– we take into account the following notation differences: $q(x) \to f_1(x)$, $\phi \to \Phi_{S_1} - (\pi/2)$;
– we identify

$$M_P \tilde{h}^f(x) = -g\frac{P}{(2\pi)^3Q}\int \mathrm{d}^2p_\perp C_S^f(x,\boldsymbol{p}_\perp), \tag{78}$$

$$M_P \tilde{f}_T^f(x) = -g\frac{P}{(2\pi)^3Q}\int \mathrm{d}^2p_\perp \delta C_S^f(x,\boldsymbol{p}_\perp), \tag{79}$$

where M_P is the proton rest mass.

5.1 Zero-order approximation

As we have seen, the left–right asymmetry at zero order vanishes. In the same approximation the muon helicity asymmetry is sensitive to the $\boldsymbol{p}_\perp$-dependent transversity distributions $\delta q^f(x_1, \boldsymbol{p}_{1\perp})$ and $\delta\overline{q}^f(x_1, \boldsymbol{p}_{1\perp})$, which can be used for calculating the transversity distributions h_1^f and $\overline{h}_1^f$ (see (29)), with $f = 1,2,3$. Equation (75) is a linear combination of the six unknown functions. In order to extract them, we need other, independent combinations. For example, considering Drell–Yan events from collisions between a polarized proton beam and, say, a pion, we get an expression for the asymmetry analogous to A_1, (75), where q^f and $\overline{q}^f$ are replaced respectively by the quark and antiquark density functions inside the pion. It is worth observing that Drell–Yan offers, at least in principle, a variety of independent combinations. For example, we could consider collisions of transversely polarized protons on unpolarized protons, antiprotons, positive and negative pions and kaons. In this way we could obtain six independent asymmetries similar to (75), from which (or from part of which) we could extract the unknown distribution functions by a fit, taking into account symmetry properties like isospin invariance, or general constraints like Soffer's inequality [53].

In this connection it is worth recalling that in [54] it was proposed, quite analogously to our work, to extract the quark helicity distributions from Drell–Yan produced in scattering on a longitudinally polarized proton target of beams of pions and unpolarized protons. These authors obtain an asymmetry formula similar to $A_1^{(0)}$, (75), although this is a twist-two and not a twist-three effect.

It is known that our procedure of convoluting the elementary cross section over the transverse momentum of the active quarks is a good approximation only for sufficiently large transverse momenta [47]. But this is not a severe constraint, since, as we have seen, transverse momentum is essential for exhibiting the helicity asymmetry. Equation (75) suggests that directions not too far from the forward and backward one are the most proper.

In order to estimate the order of magnitude of the asymmetry A_1, we take into account the HERMES results [16], i.e., $|\delta q^f|/q^f \sim |\delta\overline{q}^f|/\overline{q}^f \sim (50\pm 30)\%$. Moreover, (75) implies that A_1 should vanish for $x_1 = x_2$. Therefore, in order to maximize the asymmetry, we should take x_1 as different as possible from x_2, without making δQ^f and Q^f too small. For Q^2 of order $10\,\mathrm{GeV}^2$ and $s \sim 100\,\mathrm{GeV}^2$, a good choice, consistent with (21), is $x_1 \sim 0.5$, $x_2 \sim 0.2$, or, vice versa, $x_2 \sim 0.5$, $x_1 \sim 0.2$. The evaluation of the asymmetry is particularly complicated, because the integrand at the r.h.s. of (26) is partly positive and partly negative. We just give an upper limit: since $\overline{|\boldsymbol{p}_\perp|} \sim 0.5\,\mathrm{GeV}$, under the conditions illustrated above we estimate $|A_1| \le (10\pm 6)\%$.

5.2 First-order corrections

In order to find the order of magnitude of the first-order correction to the two asymmetries, we generalize the Qiu–Sterman [19] guess:

$$C_{S,T}^f(x,\boldsymbol{p}_\perp) = K_S q_T^f(x,\boldsymbol{p}_\perp),$$
$$C_{A,T}^f(x,\boldsymbol{p}_\perp) = K_A q_T^f(x,\boldsymbol{p}_\perp), \tag{80}$$

where $C_{S,T}^f$ and $C_{A,T}^f$ are given by (71) and (72) and K_S and K_A are constants and are independent of T. Then (67) to (70) imply

$$C_S^f(x,\boldsymbol{p}_\perp) = K_S q^f(x,\boldsymbol{p}_\perp^2),$$
$$\delta C_S^f(x,\boldsymbol{p}_\perp) = K_S \delta q^f(x,\boldsymbol{p}_\perp), \tag{81}$$

analogous relations holding true for C_A^f and δC_A^f. Two remarks are in order.

(1) The generalization (80) of the Qiu–Sterman model is quite natural and immediate in our approach, whereas it would be rather complicated in the formalism of quantum field theory.

(2) To be precise, we have generalized one of the two guesses proposed by Qiu and Sterman, the other one being obtained by multiplying the r.h.s. of (81) by x. The two models could be distinguished by examining data at very low x.

According to [19,27,55], we may set $g|K_S| = 0.08(2\pi)^3 (2^{1/2})M_P$. On the other hand, since the sums (67) and (68) consist of four terms, whereas the sums (69) and (70) consist of two terms, in order to evaluate the order of magnitude, it seems not completely unreasonable to guess that $|K_A| \simeq 1/2|K_S|$.

We fix some parameters as before: $Q^2 \sim 10\,\mathrm{GeV}^2$, $s \sim 100\,\mathrm{GeV}^2$, $x_1 = 0.5$ and $x_2 = 0.2$ (or vice versa). Moreover, we take into account once more the above mentioned results by HERMES. Lastly, as regards the angular dependence, (77) and (76) suggest that the most favorable conditions for detecting the left–right asymmetry and the first-order correction to the muon helicity asymmetry are, respectively, $\theta \sim \phi \sim \pi/2$ and $\theta \sim \pi/4, 3\pi/4, \phi = 0, \pi$. We find $|A_2| \sim (2 \pm 1)\%$ and $|A_1^{(1)}| \sim (1 \pm 0.6)\%$. The uncertainty on these two asymmetries could be even larger, if we take into account that the coupling constant g is poorly known [27]. It is worth noticing that the first-order correction to the muon helicity asymmetry is much smaller than $A_1^{(0)}$; moreover, it can be disentangled, since it exhibits a completely different angular dependence.

6 Conclusions

We have considered the muon helicity asymmetry (A_1) and the left–right asymmetry (A_2) in the Drell–Yan process $pp^\uparrow \to \mu^+\mu^- X$. The two quantities are measurable, although for A_1 some special care is required [17]. Let us recall the main results.

(1) We have calculated the two asymmetries in tree approximation by means of the improved QCD parton model [46,20], which amounts to replacing transverse momentum by a combination of momentum and gluon field, as dictated by the covariant derivative D_α (see (C.3)). Indeed both asymmetries vanish at the leading twist and turn out to be generated, at twist-three, by the transverse components of the covariant derivative, i.e.,

$$D_\alpha S^\alpha \to A_1, \quad D_\alpha \overline{S}^\alpha \to A_2.$$

In particular A_1 receives its main contribution from the first term of the covariant derivative; that is, from parton transverse momentum, while A_2 solely gets a contribution from the second term, corresponding to one gluon corrections.

(2) We have performed the calculation in the framework of the parton model, assuming an axial gauge and introducing the correlation functions between two partons of different momenta in the hadron. We have matched this simple and intuitive formalism to the quantum field theory approach, obtaining, as a result, condition (55), which uniquely fixes the normalization of the correlation functions and simultaneously guarantees gauge invariance. This procedure is of quite general validity for twist-three terms, due to the local character of the interaction.

(3) If we integrate the cross section over the transverse momentum of the muon pair, only the *effective* density and correlation functions contribute to A_1 and A_2. Therefore, it appears useful to consider these kinds of asymmetries as well as those from the differential cross section.

(4) Our approach leads to a simple physical interpretation of the "soft" functions C^f and δC^f involved in the left–right asymmetry, and to a natural generalization of the Qiu–Sterman [19] assumption, allowing for a quantitative evaluation of A_2. As a result, for standard values of Q^2 and s, this asymmetry is estimated to be a few percent.

(5) Under the same conditions the muon helicity asymmetry is probably a bit larger. This asymmetry is sensitive to the transverse momentum dependent transversity distributions, δq^f and $\delta\overline{q}^f$, which are related to the usual transversity distributions through (29). One possible advantage of our method is that one can obtain independent combinations of such functions by performing various scattering experiments with beams of unpolarized protons, pions and kaons on a transversely polarized proton target.

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Appendix A

Here we calculate the antisymmetric part of the hadronic tensor, by inserting the mass of the quark in the density matrix. We show that it reduces to the expression (24) in the limit of zero mass, the smallness of the mass being compensated by the "good" component of the Pauli–Lubanski (PL) four-vector of the active quark. The hadronic tensor reads

$$H^f_{\mu\nu} = \frac{1}{4\pi^2 Q^2}\int \mathrm{d}^2 p_{1\perp} \sum_{T_1,T_2} [q^f_{T_1}(x_1, \boldsymbol{p}_{1\perp})\overline{q}^f_{T_2}(x_2, \boldsymbol{p}_{2\perp}) \times h^{\tilde{T}_{12}}_{\mu\nu}(x_1, x_2; S) + (1 \leftrightarrow 2)], \quad (\mathrm{A.1})$$

where

$$h^{\tilde{T}_{12}}_{\mu\nu}(x_1, x_2; S) = \frac{1}{3}\mathrm{Tr}(\rho^{T_1}\gamma_\mu \overline{\rho}^{T_2}\gamma_\nu) \quad (\mathrm{A.2})$$

and

$$\rho^T(\overline{\rho}^T) = \frac{1}{2}(\not{p} \pm m_f)(1 + \gamma_5 \not{S}^T_f). \quad (\mathrm{A.3})$$

Here $p = p_1$ or p_2 and the + and − sign refer, respectively, to the quark and to the antiquark; moreover, S^T_f is the PL four-vector of the quark. We have

$$S^T_{f\alpha} = 2TL^\beta_\alpha \xi_\beta. \quad (\mathrm{A.4})$$

Here $\xi \equiv (0, \boldsymbol{S})$ is the PL four-vector of the proton, in the frame in which it is at rest. L is the matrix of the boost which takes the quark from rest to a four-momentum $p \equiv ((m_f^2+\boldsymbol{p}^2)^{1/2}, \boldsymbol{p})$, where $\boldsymbol{p} \equiv (\boldsymbol{p}_\perp, xP)$ and P is the proton momentum. A standard calculation shows that

$$S_f^T \simeq \frac{2T}{m_f x}\boldsymbol{p}_\perp \cdot \boldsymbol{S}(1,0,0,1) \;, \tag{A.5}$$

having taken the z-axis along the momentum of the polarized proton. On the other hand the antisymmetric part of the tensor (A.1) turns out to be

$$A^f_{\mu\nu} = \frac{\mathrm{i}}{4\pi^2 Q^2}\int \mathrm{d}^2 p_{1\perp} \sum_{T_1,T_2} a^f_{\mu\nu}[q^f_{T_1}(x_1,\boldsymbol{p}_{1\perp})\overline{q}^f_{T_2}(x_2,\boldsymbol{p}_{2\perp}) - (1 \leftrightarrow 2)], \tag{A.6}$$

where

$$a^f_{\mu\nu} = \frac{1}{3} m_f \epsilon_{\alpha\mu\beta\nu} S_f^{T_1\alpha} q^\beta, \tag{A.7}$$

and $q = p_1 + p_2$ is the four-momentum of the virtual photon. Taking into account (A.5), we get

$$a^f_{\mu\nu} \simeq \frac{2T_1}{3}\frac{\boldsymbol{p}_{1\perp}\cdot\boldsymbol{S}}{x_1}\epsilon_{\alpha\mu\beta\nu}\sqrt{2}n_1^\alpha q^\beta. \tag{A.8}$$

Here $p_2 \simeq x_2 P(2^{1/2})n_2$ and $n_{1(2)} \equiv (1/(2^{1/2}))(1,0,0,\pm1)$. Substituting the expression (A.8) into the antisymmetric tensor (A.6), and recalling the kinematic relations deduced in Sect. 3, $A^f_{\mu\nu}$ turns out to coincide with (24).

Appendix B

Here we derive the expression of the spin correlation matrix of two quarks; moreover, we study in detail the coupling between a transversely polarized quark and a linearly polarized gluon.

B.1 Spin correlation matrix

The spin correlation matrix is defined as

$$\kappa^{T,T'}(p,p') = u_T(p)\overline{u}_{T'}(p'), \tag{B.1}$$

where $u_T(p)$ is the Dirac spinor of a quark with four-momentum p and third spin component T. We consider a nonvanishing quark mass. First of all, we take both quarks at rest, secondly we make a Lorentz boost, thirdly we change the momentum of one of the two fermions; lastly we treat the limiting case of a negligible mass.

B.1.1 Both quarks at rest

Choosing, as usual, the z-axis as the quantization axis, we have

$$\kappa_0^{T,T'} = \sum_{i,j=1}^{2} \chi_i^T \chi_j^{T'\dagger} a_{ij} = a_{11}(1+\sigma_3) + a_{12}(\sigma_1 + \mathrm{i}\sigma_2) + a_{21}(\sigma_1 - \mathrm{i}\sigma_2) + a_{22}(1-\sigma_3), \tag{B.2}$$

where χ^T is the usual spinor. For pure spin states, such as those we are interested in, some coefficients are nonvanishing: for $T = T' a_{12}$ and a_{21} vanish, while for $T \neq T' a_{11} = a_{22} = 0$. In particular, we have

$$\kappa_0^{T,T} = m(1+2T\sigma_3), \quad \kappa_0^{T,-T} = m(\sigma_1 + 2\mathrm{i}T\sigma_2), \tag{B.3}$$

having chosen for the Dirac spinor the normalization $\overline{u}_T(p)$ $u_T(p) = 2m$, where m is the rest mass of the quark. The previous expressions can be generalized by introducing three mutually orthogonal unit vectors, $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$. Choosing the quantization axis along $\boldsymbol{k}$, we obtain

$$\sigma_3 \to \sigma_i k^i, \quad \sigma_1 + 2\mathrm{i}T\sigma_2 \to \sigma_i b_T^i, \quad \boldsymbol{b}_T = \boldsymbol{i} + 2\mathrm{i}T\boldsymbol{j}. \tag{B.4}$$

In terms of the Dirac matrices the spin correlation matrix at rest can be written as

$$\kappa_0^{T,T} = \frac{m}{2}(1\pm\gamma_0)(1 - 2T\gamma_5\gamma_i k^i),$$
$$\kappa_0^{T,-T} = -\frac{m}{2}(1\pm\gamma_0)\gamma_5\gamma_i b_T^i, \tag{B.5}$$

where m is the fermion rest mass and the $\pm$ sign in front of γ_0 refers to the (anti-)quark.

B.1.2 Quarks with equal momenta

The previous formulae can be easily extended to the case of two quarks with equal momenta. We get, similarly to the spin density matrix,

$$\kappa_p^{T,T} = \frac{1}{2}(\not p \pm m)(1 + 2T\gamma_5 \not k), \quad \kappa_p^{T,-T} = \frac{1}{2}(\not p \pm m)\gamma_5 \not b_T, \tag{B.6}$$

where, in the rest frame of the fermions, $k \equiv (0, \boldsymbol{k})$ and $b_T \equiv (0, \boldsymbol{b}_T)$.

It is worth noticing that, if the two quarks have different third spin components and momenta not parallel to the quantization axis, the final result (second formula of (B.6)) depends on the choice of the x- and y-axis. We shall solve this ambiguity below, showing that the y–z plane must be fixed in such a way to contain the quark momentum.

B.1.3 Quarks with different collinear momenta

Consider two quarks with different momenta, in a frame where they are collinear. In this case the correlation matrix reads

$$\kappa^{T,T'}(p,p') = U(p,p')\kappa_{p'}^{T,T'}. \tag{B.7}$$

Here $p \equiv (E, \boldsymbol{p})$ and $p' \equiv (E', \boldsymbol{p}')$ are the quark four-momenta in the frame considered; moreover, $U(p,p')$ is the transformation matrix corresponding to the boost which changes p' to p, i.e.,

$$U(p,p') = \frac{1}{\sqrt{1-t^2}}\left(1 + \frac{p'_i}{|\boldsymbol{p}'|}\gamma_0\gamma_i t\right), \tag{B.8}$$

where
$$t = \frac{|\boldsymbol{p}'| - |\boldsymbol{p}|}{E' + E}. \tag{B.9}$$

By using the Dirac equation, (B.7) yields
$$\kappa^{T,T'}(p,p') = \frac{1}{\sqrt{1-t^2}}\left(1 + t\frac{E' \mp \gamma_0 m}{|\boldsymbol{p}'|}\right)\kappa^{T,T'}_{p'}. \tag{B.10}$$

B.1.4 Limit of zero quark mass

This limit is trivial in the case of $T' = T$:
$$\kappa^{T,T}(p,p') = \frac{1+t}{2\sqrt{1-t^2}}\not{p}'(1 + 2T\gamma_5\not{k}). \tag{B.11}$$

On the contrary, as we have observed above, if $T' \neq T$, and if the quark momenta $\boldsymbol{p}$ and $\boldsymbol{p}'$ are not parallel to the quantization axis, i.e. to $\boldsymbol{k}$, we are faced with an ambiguity. In particular, let us consider the case when $\boldsymbol{p}$ is orthogonal to $\boldsymbol{k}$, which is of interest to us: the correlation matrix is different according as to whether we take $\boldsymbol{i}$ or $\boldsymbol{j}$ along $\boldsymbol{p}$. We show that the latter choice is correct.

Take $\boldsymbol{j}$ along $\boldsymbol{p}$. Formula (B.10) yields
$$\kappa^{T,-T}(p,p') = \frac{-1}{2\sqrt{1-t^2}}\left(1 + t\frac{E' \mp \gamma_0 m}{|\boldsymbol{p}'|}\right)(\not{p}' \pm m)\gamma_5 \times (2\mathrm{i}T\not{p}'_c m^{-1} + \not{l}), \tag{B.12}$$

where $p'_c \equiv (|\boldsymbol{p}'|, 0, 0, E')$ and $l \equiv (0, \boldsymbol{i})$. In the limit of $m \to 0$, taking into account that $\not{p}'\not{p}'_c = O(m^2/\boldsymbol{p}'^2)$, we get
$$\kappa^{T,-T}(p,p') = \frac{1+t}{2\sqrt{1-t^2}}\not{p}'\gamma_5(\mp 2\mathrm{i}T + \not{l}). \tag{B.13}$$

In the specific case that we consider in Sect. 4, we identify the four-vector l with $\overline{S}$, which is given by (4). In order to check our choice, consider two massless quarks with equal momenta. Taking the y-axis along the quark momentum, we have
$$u^{1/2} = \frac{1}{\sqrt{2}}(u_{\mathrm{R}} + u_{\mathrm{L}}), \quad u^{-1/2} = \frac{\mathrm{i}}{\sqrt{2}}(u_{\mathrm{R}} - u_{\mathrm{L}}), \tag{B.14}$$

where u_{R} and u_{L} are respectively the spinors of the right-handed and of the left-handed quark. Substituting (B.14) into (B.1), we get
$$\kappa_p^{T,-T} = -\frac{\mathrm{i}}{2}[2T(u_{\mathrm{R}}\overline{u}_{\mathrm{R}} - u_{\mathrm{L}}\overline{u}_{\mathrm{L}}) + u_{\mathrm{L}}\overline{u}_{\mathrm{R}} - u_{\mathrm{R}}\overline{u}_{\mathrm{L}}]. \tag{B.15}$$

The usual formulae for the density matrix [49] yield $u_{\mathrm{R}}\overline{u}_{\mathrm{R}} - u_{\mathrm{L}}\overline{u}_{\mathrm{L}} = \pm\not{p}\gamma_5$. On the other hand, applying the second (B.6), we get $u_{\mathrm{L}}\overline{u}_{\mathrm{R}} - u_{\mathrm{R}}\overline{u}_{\mathrm{L}} = \mathrm{i}\not{p}\gamma_5\not{l}$. Therefore, (B.15) turns out to coincide with (B.13) for $t = 0$. Now return to (B.10) and take $\boldsymbol{i}$ (instead of $\boldsymbol{j}$) along $\boldsymbol{p}$. This choice yields a result which differs from (B.13) by a factor $2\mathrm{i}T$, and it does not satisfy the self-consistency test that we have just illustrated.

Notice that in the massless case $t = (x' - x)/(x' + x)$, where x and x' are the light cone fractional momenta of the quarks; therefore the expression (B.13) is covariant. We do not take into account the quark transverse momentum – which involves a Melosh–Wigner rotation –; however, this is not requested in the twist-three approximation, which we are considering.

Lastly it is straightforward to see that in (B.11) and (B.13) we can interchange $p \leftrightarrow p'$. This property we exploit in Sect. 4.1.

B.2 Linearly polarized gluons

Now we consider a gluon emitted by one of the two protons (say proton 2) and interacting perturbatively with the active quark of the other proton. In this connection we observe that linearly polarized gluons are naturally coupled to transversely polarized quarks. In the reference frame defined above, for a gluon travelling along the y-axis, we have [49,50]
$$G^\mu_x = \frac{1}{\sqrt{2}}(G^\mu_{\mathrm{R}} + G^\mu_{\mathrm{L}}), \quad G^\mu_z = \frac{\mathrm{i}}{\sqrt{2}}(G^\mu_{\mathrm{R}} - G^\mu_{\mathrm{L}}), \tag{B.16}$$

where G^μ_x and G^μ_z denote gluon states polarized, respectively, along the x- and along the z-axis and G_{R} and G_{L} are right-handed and left-handed gluons. Color indices have been omitted for the sake of simplicity. The perturbative coupling of a massless quark with a gluon is of the type $G^\mu_{\mathrm{R}}\overline{u}_{\mathrm{R}}\gamma_\mu u_{\mathrm{R}}$ or $G^\mu_{\mathrm{L}}\overline{u}_{\mathrm{L}}\gamma_\mu u_{\mathrm{L}}$. On the other hand, we have to do with quark states like (B.14), which couple with gluon states of the type (B.16). Such couplings read
$$(\overline{u}^{T_1}\gamma_\mu u^{T_1})G^\mu_x = \frac{1}{2\sqrt{2}}(G^\mu_{\mathrm{R}}\overline{u}_{\mathrm{R}}\gamma_\mu u_{\mathrm{R}} + G^\mu_{\mathrm{L}}\overline{u}_{\mathrm{L}}\gamma_\mu u_{\mathrm{L}}), \tag{B.17}$$
$$(\overline{u}^{T_1}\gamma_\mu u^{-T_1})G^\mu_z = -\frac{2T_1}{2\sqrt{2}}(G^\mu_{\mathrm{R}}\overline{u}_{\mathrm{R}}\gamma_\mu u_{\mathrm{R}} + G^\mu_{\mathrm{L}}\overline{u}_{\mathrm{L}}\gamma_\mu u_{\mathrm{L}}). \tag{B.18}$$

Furthermore, the gluon is related to the active quarks of protons 1 and 2 by angular momentum conservation, which implies that we have to take into account the factor $F = \mathrm{sgn}\langle 1/2, T_1; 1, T_g | 1/2, T'_1\rangle = 2T_1$, where $T_g = T'_2 - T_2$ and $T'_1 = T_g + T_1$. Angular momentum conservation implies as well that for $T'_2 = -T_2$ we set $T_2 = T_1$.

Therefore the gluon polarization four-vector is $e = 2T_2 l$ for $T'_2 = T_2$ and $e = -k\delta_{T_1,T_2}$ for $T'_2 = -T_2$, where, in the reference frame defined in Sect. A.1.4, $l \equiv (0, \boldsymbol{i})$ and $k \equiv (0, \boldsymbol{k})$.

Appendix C

Here we write the hadronic tensor according to quantum field theory, up to first order in g, adopting the axial gauge $A^a \cdot n_2 = 0$, with $a = 1, ..., 8$:
$$H^{G,f}_{\mu\nu} = \frac{1}{3}\int\int \mathrm{d}^4x\mathrm{d}^4y\mathrm{e}^{-\mathrm{i}qx}$$

$$\times [\Gamma^{2f}_{\sigma\sigma'}(y)\tilde{\Gamma}^{1f}_{\alpha\beta\beta'}(x,y) + (1 \leftrightarrow 2)] \times [\gamma^\alpha S(x-y)\gamma_\mu]^{\beta\sigma}\gamma_\nu^{\beta'\sigma'}, \quad (C.1)$$

where

$$\tilde{\Gamma}^{lf}_{\alpha\beta\beta'}(x,y) = \langle P_l|\overline{\psi}^f_{\beta' i}(0)D^{ij}_\alpha(y)\psi^f_{\beta j}(x)|P_l\rangle, \quad (C.2)$$

$$D^{ij}_\alpha(y) = \mathrm{i}\partial_\alpha\delta^{ij} + gA^a_\alpha(y)\frac{1}{2}\lambda^{ij}_a, \quad (C.3)$$

$$\Gamma^{lf}_{\sigma\sigma'}(y) = \langle P_{\bar{l}}|\psi^f_{\sigma i}(0)\overline{\psi}^f_{\sigma' i}(y)|P_{\bar{l}}\rangle, \quad (C.4)$$

$$S(x-y) = \int \frac{\mathrm{d}^4q'}{(2\pi)^4}\frac{\mathrm{i}\not{q}'}{q'^2+\mathrm{i}\epsilon}\mathrm{e}^{-\mathrm{i}q'(y-x)}. \quad (C.5)$$

Two observations are in order. First of all in the above expression we have omitted the chronological products, according to the considerations by various authors [56] (see also [57]). Secondly, the covariant derivative in the expression (C.2) implies gauge invariance of the tensor (C.1). However, we have omitted the gauge "link" operators [52] in the expressions (C.2) and (C.4), since in the gauge adopted these operators can be made unity. All this simplifies a lot the calculations and immediately allows to establish symmetry properties of the correlation functions (see Sect. 4.2).

We consider the usual Fourier expansions of the massless fermion and vector boson fields, i.e.,

$$\psi^f_{\beta i}(x) = \frac{1}{(2\pi)^{3/2}} \times \sum_{T=-1/2}^{1/2}\sum_{i=1}^{3}\int\frac{\mathrm{d}^3p}{2^{1/2}|\boldsymbol{p}|^{1/2}}[c^f_{T,i}(\boldsymbol{p})u^T_\alpha(\boldsymbol{p})\chi_i\mathrm{e}^{\mathrm{i}px} + d^{f\dagger}_{T,i}(\boldsymbol{p})v^T_\alpha(\boldsymbol{p})\overline{\chi}_i\mathrm{e}^{-\mathrm{i}px}], \quad (C.6)$$

$$A^c_\alpha(y) = \frac{1}{(2\pi)^{3/2}} \times \sum_{m=1}^{2}\int\frac{\mathrm{d}^3k}{2^{1/2}|\boldsymbol{k}|^{1/2}}[a^c_m(\boldsymbol{k})e^m_\alpha(\boldsymbol{k})\mathrm{e}^{\mathrm{i}ky} + \mathrm{c.c.}]. \quad (C.7)$$

Here e^m ($m = 1, 2$) are the two different polarization four-vectors of the physical gluons: $e^1 = S$ (i.e., along the proton spin) and $e^2 = \overline{S}$ (i.e., orthogonal to the proton spin and to the proton momentum). a^c_m and $a^{c\dagger}_m$ are the creation and annihilation operators of linearly polarized gluons.

Substituting expressions (C.6) and (C.7) into (C.1), we obtain a tensor of the type

$$H^{G,f}_{\mu\nu} = H^{G,f}_{0\mu\nu} + g(H^{G,f,a}_{1\mu\nu} - H^{G,f,b}_{1\mu\nu}). \quad (C.8)$$

Here, recalling the definition

$$\sum_{i=1}^{3}\langle P_l|c^{f\dagger}_{T_l,i}(\boldsymbol{p}_l)c^f_{T_l,i}(\boldsymbol{p}_l)|P_l\rangle = q^f_{T_l}(x_l,\boldsymbol{p}^2_{l\perp}), \quad (C.9)$$

and an analogous one for antiquark densities, we find $H^{G,f}_{0\mu\nu} = H^f_{\mu\nu}$ (see (22) in the text). Moreover, we can identify $H^{G,f,a(b)}_{1\mu\nu}$ with $H^{'f,a(b)}_{\mu\nu}$ (cfr. (40)), provided we normalize the quark correlation functions according to (55) and assume a similar formula for the antiquark correlation functions. We stress that this normalization guarantees *gauge invariance*.

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